Some Experimental Consequences of Regge Poles

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The prediction of the Regge pole hypothesis concerning the ratio of real to imaginary part of the forward scattering amplitude is pointed out, and the question of how high an energy is required to test this prediction is investigated.

I.

THE conjecture that Regge poles control the asymptotic behavior of scattering amplitudes has proved quite useful in the interpretation of high-energy scattering cross sections.[1] Several of the predictions implied by this conjecture have been verified, and many more will be tested as more experimental data become available. The purposes of this paper are: (1) to call attention to the prediction which the Regge pole theory makes concerning the ratio of real to imaginary part of the forward elastic scattering amplitude, and (2) to investigate how high an energy is required to test this result and other consequences of Regge poles which involve the real parts of scattering amplitudes.

Consider an elastic-scattering process $a+b\rightarrow a'+b'$, for which the energy variable is $s=(p_a+p_b)^2$ and for which the four-momentum transfer variable is $t=(p_{a'}-p_a)^2$. We are concerned only with forward scattering, $t=0$. For definiteness let us discuss the case of pion-nucleon scattering. It is convenient to use the variable $\nu=(s-m^2-\mu^2)/2m$ introduced by Chew, Goldberger, Low, and Nambu (CGLN), which is simply the total laboratory energy of the incident pion.[2] Let $f(\nu)$ be the forward scattering amplitude in the lab system, normalized such that

$$\sigma_0\equiv(d\sigma/d\Omega)_{\mathrm{lab},t=0}=|f(\nu)|^2. \qquad (1)$$

Following CGLN, we indicate the isospin state by the superscripts $(\pm)$, where $f^{(+)}$ and $f^{(-)}$ are the non-isospin-flip and isospin-flip amplitudes, respectively. Then the Regge pole terms which are relevant to the asymptotic behavior as $\nu\rightarrow\infty$ are asymptotically of the form

$$f_n^{(\pm)}(\nu)=-\beta_n\nu^{\alpha_n}(e^{-i\pi\alpha_n}\pm1)/\sin\pi\alpha_n, \qquad (2)$$

where we have written α_n as an abbreviation for $\alpha_n(0)$, since we are here concerned only with $t=0$.

Unfortunately, there do not exist at present reliable theories which provide means of calculation of α_n and β_n. In the absence of such theories one can either attempt to extract predictions which are independent of the unknown parts of the theory or one can use Eq. (2) as the basis of a phenomenology of high-energy scattering. We wish to call attention here to an implication of Eq. (2) which may be useful for these purposes; namely, that the ratio $\mathrm{Re}f_n/\mathrm{Im}f_n$ is energy-independent, and is given by

$$\begin{aligned}\mathrm{Re}f_n^{(+)}(\nu)/\mathrm{Im}f_n^{(+)}(\nu)&=-\cot(\pi\alpha_n/2),\\ \mathrm{Re}f_n^{(-)}(\nu)/\mathrm{Im}f_n^{(-)}(\nu)&=\tan(\pi\alpha_n/2).\end{aligned} \qquad (3)$$

These relations provide a method of measuring the parameters α_n. The only information currently available about these parameters comes from measurements of energy dependence of total cross sections, and is at present quite imprecise.[3,4]

As our first example let us consider the pion-nucleon isospin-flip forward amplitude, $f^{(-)}(\nu)$. Udgaonkar[3] has pointed out that the asymptotic behavior of this amplitude is controlled by ρ-meaon exchange, so we find

$$\lim_{\nu\rightarrow\infty}\mathrm{Re}f^{(-)}(\nu)/\mathrm{Im}f^{(-)}(\nu)=\tan(\pi\alpha_\rho/2). \qquad (4)$$

The quantity $\mathrm{Im}f^{(-)}(\nu)$ is related to total cross sections by the equation $\mathrm{Im}f^{(\pm)}=(\nu^2-1)^{1/2}\sigma_T^{(\pm)}(\nu)/4\pi$, where

$$\sigma_T^{(\pm)}(\nu)=\tfrac{1}{2}[\sigma_T^{\pi^-p}(\nu)\pm\sigma_T^{\pi^+p}(\nu)]. \qquad (5)$$

The real parts can be found to within a sign ambiguity by extrapolating angular distributions to $t=0$; for example,

$$(\mathrm{Re}\langle\pi^+p|f|\pi^+p\rangle)^2=\sigma_0^{\pi^+p}-\left[\frac{(\nu^2-1)^{1/2}}{4\pi}\sigma_T^{\pi^+p}\right]^2, \qquad (6)$$

where $\langle\pi^+p|f|\pi^+p\rangle$ is the forward scattering amplitude for $\pi^+p\rightarrow\pi^+p$. Then one finds

$$f^{(\pm)}=\tfrac{1}{2}[\langle\pi^-p|f|\pi^-p\rangle\pm\langle\pi^+p|f|\pi^+p\rangle], \qquad (7)$$

and also

$$f^{(-)}=-(1/\sqrt{2})\langle\pi^0n|f|\pi^-p\rangle. \qquad (8)$$

Thus, an experimental determination of α_ρ is possible in principle via Eqs. (4)–(8). We return in the second half of this paper to the question of the corrections which should be made if Eq. (4) is used at finite energies.

Since the $f^{(+)}$ amplitude is dominated by the Pom-

[1] T. Regge, Nuovo Cimento **14**, 951 (1959); **18**, 947 (1960). G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961); **8**, 41 (1961); G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. **126**, 1202 (1962); S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, *ibid.* **126**, 2204 (1962); R. Blankenbecler and M. L. Goldberger, *ibid.* **126**, 766 (1962).

[2] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

[3] B. M. Udgaonkar, Phys. Rev. Letters **8**, 142 (1962).

[4] G. von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters **8**, 173 (1962).

eranchuk trajectory, it follows that

$$\lim_{\nu\to\infty} \mathrm{Re}f^{(+)}(\nu)/\mathrm{Im}f^{(+)}(\nu) = -\cot(\pi\alpha_P/2). \quad (9)$$

The fact that this ratio is experimentally small supports, of course, the conjecture that $\alpha_P=1$.[1,6] Accepting this conjecture one can proceed to measure α for the P' trajectory introduced by Igi, who showed that the existence of a second trajectory with quantum numbers of the vacuum and $\alpha_{P'}\approx 0.5$ is necessary for the consistency of the forward dispersion relations.[7] One finds

$$\lim_{\nu\to\infty} \mathrm{Re}f^{(+)}(\nu)/\mathrm{Im}f^{(+)\prime}(\nu) = -\cot(\pi\alpha_{P'}/2), \quad (10)$$

where

$$\mathrm{Im}f^{(+)\prime}(\nu) = \mathrm{Im}f^{(+)}(\nu) - \nu\sigma_T^{(+)}(\nu)/4\pi. \quad (11)$$

These equations permit an experimental determination of $\alpha_{P'}$.

II.

The usefulness of the equations written down above is diminished by their being exact only at infinite energies. The current Regge pole theory of very high-energy reactions does not make firm statements about how large an error is made by working at a given finite energy. In the absence of such statements an empirical approach has been adopted. The attempt is being made to fit the high-energy data above the resonance region in terms of a few Regge poles. If this procedure is successful it will be regarded as evidence for the dominance of Regge poles at these energies. Already the pion-nucleon total cross sections and the nucleon-nucleon angular distributions have been shown to be well represented in terms of a few Regge poles.[4,6] The equations derived above for the ratio Ref/Imf could be used in a similar empirical way. It is, however, possible to make some further theoretical progress in the case of pion-nucleon scattering by using the forward scattering dispersion relations.

Let us examine the dispersion relations for the forward, isospin-flip, pion-nucleon scattering amplitude. The asymptotic behavior of this amplitude is, according to Eq. (2),

$$f^{(-)}(\nu) \sim i\beta_\rho \nu^{\alpha_\rho}. \quad (12)$$

It is expected theoretically and verified experimentally that $\alpha_\rho<1$.[3,4] This permits one to write a Cauchy integral for $f^{(-)}(\nu)/\nu$, obtaining the familiar relation

$$f^{(-)}(\nu) = \frac{2\nu f^2}{\nu^2-\nu_B^2} + \frac{\nu}{2\pi^2}\int_1^\infty d\nu' \frac{(\nu'^2-1)^{1/2}\sigma^{(-)}(\nu')}{\nu'^2-\nu^2}, \quad (13)$$

where the pion mass has been set equal to unity, and where $\nu_B^2=1/4m^2$. Let us then define the following function

$$f_\rho(\nu) = -\beta_\rho \frac{P_{\alpha_\rho}(-\nu)-P_{\alpha_\rho}(\nu)}{\sin\pi\alpha_\rho}, \quad (14)$$

which has the same asymptotic form as $f^{(-)}(\nu)$ and which has branch points in the same places. Performing a Cauchy integral one finds for $f_\rho(\nu)$ the representation

$$f_\rho(\nu) = \frac{2\nu\beta_\rho}{\pi}\int_1^\infty d\nu' \frac{P_\alpha(\nu')}{\nu'^2-\nu^2}. \quad (15)$$

One can then subtract Eq. (15) from Eq. (13) to obtain the following convenient form:

$$f^{(-)}(\nu) = f_\rho(\nu) + F^{(-)}(\nu), \quad (16)$$

where

$$F^{(-)}(\nu) = \frac{2\nu f^2}{\nu^2-\nu_B^2} + \frac{\nu}{2\pi^2}\int_1^\infty \frac{d\nu'}{\nu'^2-\nu^2} \times[(\nu'^2-1)^{1/2}\sigma^{(-)}(\nu') - 4\pi\beta_\rho P_{\alpha_\rho}(\nu')]. \quad (17)$$

The subtracted form of the dispersion relations given by Eqs. (16) and (17) is convenient because it separates the term $f_\rho(\nu)$, which is responsible for the asymptotic behavior, from the correction term $F^{(-)}(\nu)$. This term will be dominated by the resonance region, since the factor in brackets in Eq. (17) is constructed to vanish as $\nu \to \infty$. We adopt the hypothesis that the total cross section is given in terms of Regge poles above an energy ν_c, so that we can let the upper limit on the integral in Eq. (17) be ν_c. The data are consistent with $\nu_c \geq 32$ (4 BeV).[4]

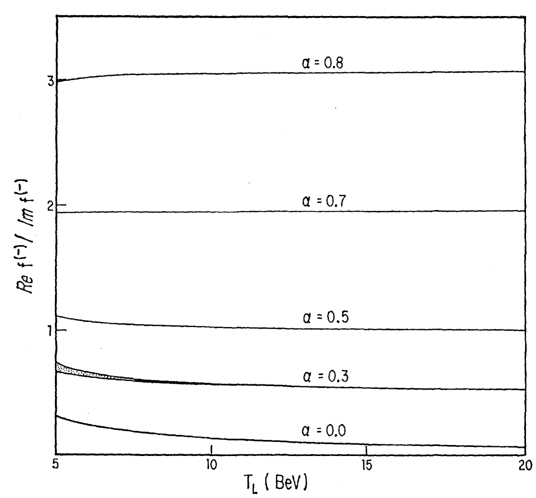

FIG. 1. The ratio Re$f^{(-)}$/Im$f^{(-)}$ as calculated from the forward dispersion relations, for the various sets of $\alpha_\rho(0)$ and $\beta_\rho(0)$ given by von Dardel *et al.*[4]

[5] It is perhaps worth mentioning that if one measures the total cross section at high energies and finds that it fits the form $\sigma_T \propto \nu^{\alpha(0)-1}$, then the use of the forward dispersion relations to determine Ref results in Eq. (4) being satisfied identically. To obtain an independent determination of $\alpha(0)$ one must measure Ref directly.

[6] A. Diddens, E. Lillethun, G. Manning, A. Taylor, T. Walker, and A. Wetherell, Phys. Rev. Letters 9, 108, 111 (1962).

[7] Keiji Igi, Phys. Rev. Letters 9, 76 (1962).

A numerical evaluation of the dispersion integral yields the curves shown in Fig. 1 for $\mathrm{Re}f^{(-)}/\mathrm{Im}f^{(-)}$ for the sets of β_ρ and α_ρ which von Dardel *et al.* find give acceptable fits to the pion-nucleon total cross sections from 5 to 20 BeV.[4] The cutoff has been set at $\nu_c=32.1$ (4.2 BeV). The two curves for $\alpha=0.3$ represent an estimate of the uncertainty in the dispersion integral. The results at lower energies have not been shown because of their sensitivity to small changes in the experimental data used in evaluating the dispersion integral.

The different sets of $\alpha\rho$ and $\beta\rho$ give very different values of $\mathrm{Re}f^{(-)}(\nu)/\mathrm{Im}f^{(-)}(\nu)$. At all energies shown, the contribution of the correction term $F^{(-)}(\nu)$ is small compared to the difference between the different sets of α_ρ and β_ρ. A sufficiently accurate measurement of $\mathrm{Re}f^{(-)}(\nu)/\mathrm{Im}f^{(-)}(\nu)$ at an energy above 5 BeV will yield information about the parameter $\alpha_\rho(0)$. The accuracy required can be determined from Fig. 1. The curves are insensitive to the value of β_ρ, since the asymptotic value of $\mathrm{Re}f/\mathrm{Im}f$ is independent of β_ρ. It affects only the deviation of the curves from constant behavior, which can be seen from the figure to be a small effect compared to the separation in the curves corresponding to different $\alpha_\rho(0)$.

III.

Another interesting point emerges from Fig. 1: The correction term $F^{(-)}(\nu)$ can be significant well above the energy at which the total cross section is well represented in terms of Regge poles, as evidence by the deviation from constancy of the curves in Fig. 1. That is, the corrections to $\mathrm{Re}f^{(-)}$ persist to a much higher energy than do the corrections to $\mathrm{Im}f^{(-)}$. This is due to the competition of narrow low-energy resonances with the asymptotic Regge terms. If a resonance is represented by

$$f(\nu)=\Gamma/(\nu_r-\nu+i\Gamma),$$

then for $\nu\gg\nu_r$,

$$\mathrm{Re}f(\nu)\sim\Gamma/(\nu_r-\nu)$$

whereas

$$\mathrm{Im}f(\nu)\sim\Gamma/(\nu_r-\nu)^2.$$

Thus, the influence of a resonance dies away with energy much more slowly for $\mathrm{Re}f$ than for $\mathrm{Im}f$. This may account for the difficulties experienced by Ting, Jones, and Perl in fitting $\pi-p$ data in the 3–5 BeV range with Regge poles.[8] The comparative success of the fits to $p-p$ data may be due to the apparent absence of resonances in this process.

IV. SUMMARY

Let us recapitulate briefly. In Sec. I we called attention to the prediction made by the Regge pole theory concerning the ratio $\mathrm{Re}f/\mathrm{Im}f$, namely, that a measurement of this ratio at "asymptotic" energies would determine the parameter $\alpha(0)$ of the Regge pole concerned. In Sec. II we used the forward scattering dispersion relations to estimate the magnitude of the correction terms (not arising from the Regge pole) which enter at finite energies. It was shown that at energies greater than 5 BeV the ratio $\mathrm{Re}f/\mathrm{Im}f$ is very sensitive to $\alpha(0)$, and is insensitive to uncertainties in the parameter β and in the experimental data in the resonance region. In Sec. III the observation was made that in reactions where resonances are present $\mathrm{Re}f$ approaches its asymptotic form more slowly as a function of energy than does $\mathrm{Im}f$. Thus, it should be possible to fit nucleon-nucleon angular distributions with a Regge pole formula at a lower energy than that for the pion-nucleon case.[8]

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APPENDIX: IMPLICATIONS OF CUTS IN COMPLEX ANGULAR MOMENTUM PLANE

Several authors have noted the possibility of cuts in the complex angular momentum plane.[9] Recently, Mandelstam has given strong arguments in support of their existence.[10] Such cuts would impose some modifications on the arguments given in this paper.

Let us consider, for example, the $f^{(-)}(\nu)$ amplitude for forward pion-nucleon scattering. The asymptotic behavior would be controlled by, in addition to the contribution of the ρ-meson Regge pole, the contribution of a cut associated with exchange of a ρ-meson and a vacuum particle; i.e.,

$$f^{(-)}(\nu)\sim f_\rho(\nu)+f_c(\nu),$$

where $f_\rho(\nu)$ was defined in Eq. (14), and where[9]

$$f_c(\nu)\sim\int_{\alpha_{\min}}^{\alpha_\rho} dl\,\nu^l g(l)[\tan(\pi l/2)+i].$$

Asymptotically, we find

$$\mathrm{Re}f^{(-)}(\nu)\sim\beta_\rho\nu^{\alpha_\rho}\tan(\pi\alpha_\rho/2)\left[1+\frac{g(\alpha_\rho)}{\beta_\rho\ln\nu}+O\left(\frac{1}{\ln^2\nu}\right)\right],$$

$$\mathrm{Im}f^{(-)}(\nu)\sim\beta_\rho\nu^{\alpha_\rho}\left[1+\frac{g(\alpha_\rho)}{\beta_\rho\ln\nu}+O\left(\frac{1}{\ln^2\nu}\right)\right].$$

Therefore,

$$\mathrm{Re}f^{(-)}(\nu)/\mathrm{Im}f^{(-)}(\nu)\sim\tan(\pi\alpha_\rho/2)[1+O(1/\ln^2\nu)].$$

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[8] C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).

[9] D. Amati, S. Fubini, and A. Stanghellini, Physics Letters **1**, 29 (1962); CERN Report TH 264. R. Blankenbecler, Bull. Am. Phys. Soc. **7**, 58 (1962).

[10] S. Mandelstam (unpublished).

Thus, the statements in Sec. I remain valid, but the rate of approach to the asymptotic limit, discussed in Sec. II, must be corrected by the terms of order $1/\ln^2\nu$. Lacking any method for estimating the size of these terms, we must rely on experiment to determine the energy at which asymptotic forms can be used. In particular, one can apply the test proposed in this paper: Measure $\mathrm{Re}f^{(-)}/\mathrm{Im}f^{(-)}$ at several energies to check whether it is constant. If it is, its ratio is simply $\tan(\pi\alpha_\rho/2)$, as in the case of no branch cuts.

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Classical Radiation Recoil. II*

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The angular momentum carried off by electromagnetic or gravitational waves is evaluated by computing the torque exerted by the radiation on the emitting system. The lowest order secular effect in the electromagnetic case is proportional to the vector product of the first and second time derivatives of the electric dipole. In the gravitational case, the lowest order term is proportional to the antisymmetrized product of the second and third time derivatives of the mass quadrupole.

IT is well known that angular momentum can be carried off by electromagnetic waves[1] and presumably also by gravitational waves. The purpose of this paper is to apply the previously developed technique[2] to a direct calculation of the torque exerted on the emitting system by its own electromagnetic or gravitational radiation. This method seems physically more meaningful than discussions about what is happening at spatial infinity.

The torque is given by

$$N^k=\epsilon^{kmn}\int x_m F_n dV, \qquad (1)$$

where F_n is the force density. In the electromagnetic case, we have

$$F_n=(A_{\alpha n}-A_{n\alpha})J^\alpha. \qquad (2)$$

With the help of the continuity equation $J_\alpha{}^\alpha=0$, it follows that

$$N^k=\epsilon^{kmn}\int (x_m A_{\alpha n}J^\alpha+A_n J_m)dV, \qquad (3)$$

and substitution of (A12), (A13), and repeated use of (A14) readily lead to

$$N^k=\tfrac{4}{3}\epsilon^{kmn}\,{}^1D_m\,{}^2D_n, \qquad (4)$$

in agreement with the usual result.[1]

In the gravitational case, we have[2]

$$F_n=\tfrac{1}{2}(V_{n\alpha\beta}+V_{n\beta\alpha}+\tfrac{1}{2}g_{\alpha\beta}V^\gamma{}_{\gamma n}-V_{\alpha\beta n})T^{\alpha\beta}. \qquad (2')$$

With the help of the dynamic equation $T^{\alpha\beta}{}_\beta=0$, it follows that

$$N^k=\epsilon^{kmn}\int[\tfrac{1}{2}x_m(\tfrac{1}{2}g_{\alpha\beta}V^\gamma{}_{\gamma n}-V_{\alpha\beta n})T^{\alpha\beta}-V_{n\alpha}T^\alpha{}_m]dV, \qquad (3')$$

and substitution of (A12′), (A13′), and repeated use of (A14′) and (A14″) readily lead to

$$N^k=\tfrac{2}{5}\epsilon^{kmn}\,{}^3Q_{mj}\,{}^2Q_{jn}. \qquad (4')$$

It would be interesting to rederive this result by computing the angular momentum flow, in the wave zone, of the gravitational radiation, as recently done by Papapetrou for the linear momentum flow.[3] This seems, however, to be an extremely tedious calculation, much more difficult than in the electromagnetic case.[1]

It should also be noted that radiation recoil is a secular (cumulative) effect. It might, therefore, be easier to detect than the instantaneous gravitational radiation energy flow.

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[1] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 254.
[2] A. Peres, Phys. Rev. **128**, 2471 (1962), hereafter referred to as A. The notations of A are used throughout.
[3] A. Papapetrou, Compt. Rend. **255**, 1578 (1962).